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# Drone Squadron Optimization: a Self-adaptive Algorithm for Global Numerical Optimization

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**Abstract**—This paper proposes Drone Squadron Optimization, a new self-adaptive metaheuristic for global numerical optimization which is updated online by a hyper-heuristic. DSO is an artifact-inspired technique, as opposed to many algorithms used nowadays, which are nature-inspired. DSO is very flexible because it is not related to behaviors or natural phenomena. DSO has two core parts: the semi-autonomous drones that fly over a landscape to explore, and the Command Center that processes the retrieved data and updates the drones' firmware whenever necessary. The self-adaptive aspect of DSO in this work is the perturbation/movement scheme, which is the procedure used to generate target coordinates. This procedure is evolved by the Command Center during the global optimization process in order to adapt DSO to the search landscape. DSO was evaluated on a set of widely employed benchmark functions. The statistical analysis of the results shows that the proposed method is competitive with the other methods in the comparison, the performance is promising, but several future improvements are planned.

## I. INTRODUCTION

Meta-heuristics are general optimization methods used to solve a variety of problems [1]. When derivatives are costly or impossible to obtain, derivative-free methods such as meta-heuristics are usually employed for global optimization. Those methods make few or no assumptions about the problem being optimized, being able to deal with problems that are non-differentiable, irregular, noisy, or dynamic. Otherwise, if derivatives are feasible, and one has a convex problem, then convex optimization methods may be orders of magnitude faster and more accurate. Many meta-heuristic algorithms are nature-inspired, with Genetic Algorithm (GA) [2] being one of the most used. Another important category of meta-heuristic is the Swarm Algorithm, introduced with the Ant Colony Optimization [3] for combinatorial optimization and the Particle Swarm Optimization (PSO) algorithm in 1995 [4] to solve continuous optimization problems as in this work.

PSO has inspired the creation of many swarm intelligence algorithms, such as Bees Algorithm, Artificial Bee Colony, Grey Wolf Optimizer, Firefly algorithm, Bat algorithm, Gravitational Search Algorithm, Glowworm swarm optimization, Cuckoo Search, among others. Fister Jr et al. [5] provide a short review and a list of nature-inspired methods.

Here we change the source of inspiration. Instead of adopting a paradigm from nature, we propose an *artifact-inspired algorithm*<sup>1</sup>, that is, it is inspired by something artificially created (human-made), more specifically, *drones*. In this paradigm, such an algorithm is not bound by a particular realization at hand as drones are flexible machines (at several aspects), not biological entities. Instead, it can use a variety of different mechanisms/procedures without losing its core characteristics.

Another important aspect of the technique proposed here is that it is self-adaptive regarding code modification, not only in the parameters configuration. This means that the technique can change the procedures that the drones use to generate solutions, meaning it can partially evolve itself during the search. Some researchers did similar work, but there are important differences that are shown later in this paper.

Self-adaptation is one of the characteristics that can provide largest improvements in performance [6]–[8]. However, most techniques use human-developed adaptation schemes that do not cover every problem and may be unable to perform well on dynamic situations. Thus, methods that can learn and self-adapt are of great value. The key contributions of this paper are:

- the proposal of an artifact-inspired paradigm that is not tied to any natural phenomena or behavior but could automatically act like any of them;
- a novel self-adaptive metaheuristic that can evolve itself on-the-fly, transforming its behavior into that of other paradigms such as evolutionary or swarm;
- an explicit separation between the Controller and the semi-autonomous exploration entities in a team approach.

The remaining of the paper is structured as follows: Section II presents our proposal. In Section III we present related works. Section IV demonstrates the strength of the method with computational experiments. A further discussion is given in Section V. Section VI presents the conclusions and future work.

<sup>1</sup>The terminology employed in this work is using the artifact as a metaphor which - by way of analogy - can facilitate ones understanding.

## II. DRONE SQUADRON OPTIMIZATION

Drones, like the artificially built submarine machines or the well-known flying machines, such as balloons, airplanes, helicopters, quad-copters, can navigate autonomously or remotely, have sensors, can communicate over large distances, can use solar power energy, and, one of the most important features: can be upgraded/improved not only in terms of hardware but also by changing their software (the firmware). Therefore, as the artificially built machine has a software (firmware) to control its behavior, researchers are free to add mechanisms to the algorithm as common software upgrades, which is easier than looking for a natural phenomena to justify the improvement.

The Drone Squadron Optimization (DSO) proposed here may be related to Particle Swarm Optimization, Artificial Bee Colony, or any other Swarm algorithm, because it is based on the movement of entities in search-space. However, as explained before, the movement of the squadron is not necessarily based on behavior observed in nature. DSO's approach allows it to automatically choose to use recombination and/or perturbation of solutions with distinct procedures, making it act as an evolutionary algorithm, swarm algorithm, probabilistic algorithm, or other, according to how it performs on the search landscape. Moreover, those procedures may have their *actual code* updated during the search.

The DSO algorithm presented here is composed of a Drone Squadron with different teams and a Command Center, which uses information collected by/from the drones to maintain partial control of the search, and to develop new firmware for controlling the drones (see Figure 1). A drone *is not* a solution; it moves to a coordinate which is a solution. The firmware contains the procedures (codes) and configurations used by the teams to search the landscape. In this work, the perturbation procedure is an actual source code, a *string* to be parsed and executed by the drone.

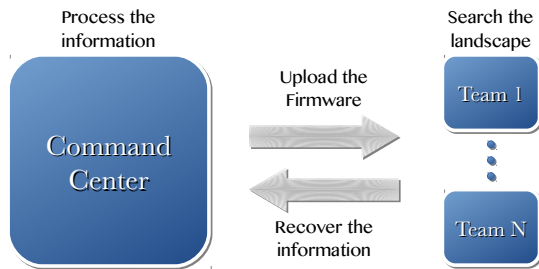


Figure 1. High-level DSO abstraction.

Conceptually, the Command Center is a central place for carrying out orders and for supervising tasks. It receives inputs, processes data, and generates outputs from decisions made. The Command Center can update the firmware of drones whenever it decides, dynamically adapting drone behavior to the problem.

A group of drones is divided into teams of the same size (necessary to the selection mechanism), where each team has its firmware that controls the movement of each drone; there

are, thus, distinct firmware for each team. This means that *each team has a distinct way of sampling the search space from the same current set of solutions*; the teams are not like species nor niching in evolutionary algorithms.

The drones have a search mission on the landscape (the objective function) to locate a particular target, whose value is obtained by the drone's sensor. The teams are not necessarily multiple groups searching distinct and distant regions of the landscape. In fact, all drones move from specific departure points that may be the same for some of the teams, but not for others. As the teams have distinct firmwares, even though they move from the same departure points they may get to distinct coordinates. Also, for the same reason teams may overlap search regions. Moreover, a team does not have to follow another team, unless the Command Center encode that in the firmware.

Algorithm 1 presents a high-level algorithm of DSO, while the next subsections explain more details. One may observe that DSO is more complex than traditional nature-inspired methods as it has several other characteristics to deal with the Control Center, the teams, the firmware adaptation, among others.

### A. Drone movement

The drones use an autonomous system to calculate target positions, move to them, and collect the information that is sent back to the Command Center. The following mechanisms available to DSO are employed in various optimization techniques, either evolutionary or non-evolutionary. Each mechanism may have more than one implementation giving DSO many exploration and exploitation capabilities.

For the current DSO version, the firmware contains only the mechanism to generate new trial coordinates (TC) through perturbation; thus, it is the core of the drone's firmware. New firmware are generated based on the well-known perturbation scheme of a biased random walk:

$$P = \text{Departure} + \text{Offset}(), \quad (1)$$

$$TC = \text{calculate}(P), \quad (2)$$

where *Departure* is a coordinate (a solution point in the search space), *Offset* is a function that returns the perturbation movement (a numerical value), and *P* is the complete perturbation formula that has to be calculated to return the trial coordinates<sup>2</sup>. This pattern of moving from a *Departure* point is important because it avoids the algorithm to simply shrink the search space towards the origin<sup>3</sup>. In fact, even though the *Offset* is shrinking to zero, it is still a noise applied to the *Departure* coordinates, which results in a neighborhood search.

Instead of using a hard-coded procedure, *P* is automatically updated during the global optimization process and adapted to

<sup>2</sup>It is important to notice that all solutions are one-dimension arrays; therefore, all operations present in this work are element-wise.

<sup>3</sup>Many well-known benchmark functions have their global optimum at the origin and there are algorithms that exploit such characteristic to achieve high performance.

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**Algorithm 1** High-level DSO algorithm. The abbreviations are: *GlobalBestCoords* (GBC), *GlobalBestOFV* (GBOFV), *CurrentBestCoords* (CBC), *CurrentBestOFV* (CBOFV), *TeamCoords* (TmC), *TeamOFV* (TmOFV), *TrialCoords* (TC).

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1: Input: objective function, problem bounds ( $\overrightarrow{LB}, \overrightarrow{UB}$ ), user defined constants, number of teams ( $t$ ), number of drones per team ( $N$ ),
   maximum number of iterations ( $MaxIt$ ), maximum number of iterations without improvement (MaxStagnation), options for the firmware
2: Output: Best solution (GBC) and best Objective function value (GBOFV)
3:
4: Initialize the Command Center, drones and teams in TeamCoords, where TeamCoords are sets of teams containing the drones'
   coordinates
5: Send the drones to scan the landscape at  $(t \times N)$  random coordinates
6: Select the  $N$  best coordinates to be CBC
7:
8: while stopping criteria are not met do
9:
10:    for each team  $\in [1, 2, \dots, t]$  do
11:        for each drone  $\in [1, 2, \dots, N]$  do
12:            Generate TC using the perturbation scheme of the current team
13:            Choose the recombination method and apply it to combine TC with  $CBC_{drone}$ , resulting in the  $TmC_{drone,team}$ 
14:            Choose out-of-boundary correction method and apply it to  $TmC_{drone,team}$ , saving the violations that occurred
15:            Move to new coordinate ( $TmC_{drone,team}$ ), scan the landscape, and set  $TmOFV_{drone,team}$  = objective function value
16:        end for
17:    end for
18:
19:    Select the best coordinates found by the teams in the current iteration
20:    Set improved = 0
21:    for each drone  $\in [1, 2, \dots, N]$  do
22:        Rank  $TmC_{drone,team}$  according to  $TmOFV_{drone,team}$ 
23:        Set bestIdx = index of the best  $TmOFV_{drone,team}$ 
24:        if  $TmOFV_{drone,bestIdx} < CBOFV_{drone}$  then
25:            Set  $CBOFV_{drone} = TmOFV_{drone,bestIdx}$ 
26:            Set  $CBC_{drone} = TmC_{drone,bestIdx}$ 
27:            if  $TmOFV_{drone,bestIdx} < GBOFV$  then
28:                Set improved = improved + 1
29:                Set  $GBOFV = TmOFV_{drone,bestIdx}$ 
30:                Set  $GBC = TmC_{drone,bestIdx}$ 
31:            end if
32:        end if
33:    end for
34:    if improved == 0 then
35:        Check and apply stagnation control
36:    end if
37:    Set TeamQuality as the average rank plus boundary violations
38:    Update the firmware, considering TeamQuality
39: end while

```

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the problem. The adaptation will be explained further as it is not performed by the drone itself. Both the *Departure* coordinates and the function that gives the *Offset* are modified, that is, distinct teams may have distinct ways of choosing departure coordinates and how to calculate the offset. As examples, suppose the two following perturbations for teams 1 and 2, where  $C_1$  is a user-defined constant,  $G(0,1)$  is a scalar sampled from a Gaussian distribution with zero mean and unit standard deviation, and  $U(0,1,D)$  is an array of  $D$  columns of numbers sampled from a Uniform distribution with minimum 0 and maximum 1. The meaning of the terms can be seen in Algorithm 1.

$$P_1 : GBC + (C_1 \times (GBC - CBC_{drone})). \quad (3)$$

$$P_2 : CBC_{drone} + (G(0,1) \times (\sqrt{U(0,1,D)} + CBC_{drone})). \quad (4)$$

As one may observe, the two *Departures* are the arrays  $GBC$  and  $CBC_{drone}$ , while the added expressions are the *Offsets*. The perturbation scheme in DSO was implemented using a *tree-structure* representation with terminal and non-terminal nodes, and always follows the pattern shown in Equation 1, that is, a sum of two terms where the first one is selected from a particular subset of the terminals, and the second one is an expression grown using the available terminals and non-terminals.

Initially, the perturbation schemes were randomly generated (arbitrary equations). However, after preliminary experiments it was identified that a completely random set of initial

perturbation schemes shows poor performance. To overcome this issue, a set of *reference perturbations* was defined, that is, equations that will be used as *initial* perturbations for the teams, but that may be replaced during the optimization process.

When exploiting a particular region of the landscape, the drones may generate identical or almost identical target coordinates. Convergence avoidance mechanisms are used to help the drones escape from local optima. The use of scaling (Gaussian, Uniform, etc.), stagnation detection and its treatment, and generation of opposite coordinates allows movements to regions far from the neighborhood. Unfortunately, this may also slow down reaching the optimum solution and/or reduce its accuracy when a small budget is considered, but we chose to expand the exploration capability of the method to locate promising regions in the search space.

After the perturbation step generates the trial coordinates for each drone ( $TC_{drone}$ ), either a recombination to generate  $TmC$  is performed with the best coordinates found so far or no recombination is done ( $TmC = TC$ ). Currently, this selection is random and all recombination procedures available to the drones have the same probability of selection. Also, recombination is performed after perturbation, but changing the order is a perfectly plausible option. That will change the behavior of the method without invalidating the original artifact inspiration.

Finally, the drones may be allowed to move only inside a particular perimeter. Therefore, if coordinates in  $TmC$  are outside such perimeter, then a correction must be made. As in the recombination, some correction procedures are available to be chosen, and there is no bias in the current DSO.

## B. Command Center

By using the information collected by the drones, the Command Center develops and updates new firmware to upload to them. The drones use the new firmware and all data distributed by the Command Center to control their behavior. Conceptually, cheap data analysis may be performed on the drones due to their limited computing power, whereas the expensive ones are done on the Command Center and shared with the drones.

1) *Firmware update*: The Command Center uses two information to measure the performance of a team: 1) its rank regarding the objective function value, and 2) the degree of out-of-bound coordinates that were generated. It is important to consider the violations because good solutions may be generated by the correction procedures just by chance, while  $TmC$  had, in fact, big violations.

Given that each team generates  $N$  solutions, and that the ID of a drone in a *team* is its index  $drone = 1, \dots, N$ , the rank for each ID is obtained by sorting  $TmOFV_{drone,team}$  of distinct teams. For an example with three teams, sort  $TmOFV_{drone,1}$ ,  $TmOFV_{drone,2}$ ,  $TmOFV_{drone,3}$ .

Regarding the violations, they are calculated as:

$$violation_{drone,team} = \sum_{j=1}^D \left\{ \begin{array}{l} |TmC_{drone,team,j} - UB_j| \\ + \\ |LB_j - TmC_{drone,team,j}| \end{array} \right\}, \quad (5)$$

where  $D$  is the problem's dimension, and considering only the cases where  $TmC_{drone,team,j} > UB_j$  or  $TmC_{drone,team,j} < LB_j$ . The rank is calculated at every iteration and can be accumulated or averaged after a series of iterations.

As soon as a firmware updating criterion is reached (amount of iterations, for instance), the Command Center replaces the  $w$  worst firmware by variations of the  $w$  best firmware, i.e., for  $w = 1$  the team with worst accumulated rank has its firmware updated with a variant (random sub-tree replacement) of the best team firmware. There is no recombination of codes. The new variant has to satisfy the following rules:

- 1) The size  $S$  of the new perturbation  $P_k$ , where  $S$  is the number of nodes in the tree data structure, and  $k$  is the index of the worst of the  $t$  teams, has to be  $S(P_k) > s_{min}$  and  $S(P_k) < s_{max}$ , where  $s_{min}$  and  $s_{max}$  are user defined parameters;
- 2) The new perturbation has to be distinct from the original one, but the current version only detects syntactic differences, not semantic ones;
- 3) A function is not allowed to receive the same argument for the two parameters, for instance,  $sub(Shift, Shift)$ ;
- 4) The  $w$  reference perturbations must not be replaced, thus they are *fixed perturbations*;
- 5) The perturbation scheme of Equation 1 must hold.

Consequently, one expects performance improvement after replacing the firmware that gave the worst results by a variation of the firmware that gave the best results. With respect to the *fixed* perturbations, this mechanism is to provide at least one firmware with reasonable search capability. A *fixed* perturbation may be one that favors exploration, while the remaining one may be free to perform exploitation.

2) *Selection for next iteration, stagnation detection and treatment*: After all drones return their findings on the landscape (the objective function values), the Command Center must decide which information is important to keep in the search plan in order to be used in the future. A hard selection mechanism (as shown in Algorithm 1) chooses the best between the current solution and the new one, considering the objective function values. However, stagnation must be treated.

Stagnation is detected when the objective function value of the current best solution at a particular index remains the same after a certain number of iterations. When detected, a mechanism to allow further exploration is used and soft selection is performed:

```
if ( $TmOFV_{drone,bestIdx} < CBOFV_{drone}$  or  $U(0,1) < P_{acc}$ )
then  $CBC_{drone} = TmC_{drone,bestIdx}$ ,
```

where  $bestIdx$  is the index of the best drone (rank 1 considering all teams), and  $P_{acc}$  is the probability of accepting a worse

solution. Thus, some coordinates that resulted in lower-quality solutions are inserted in the search plan, replacing higher-quality solutions. Elitism concept is applied here to keep the best solution in *CBC*.

### III. RELATED WORK

Here we consider only the most closely related work, i.e. work that employs some mechanism to evolve a meta-heuristic regarding the mutation/perturbation operator. One may divide the research into two basic evolutionary branches: offline and online. In offline evolution, the main method is a hyper-heuristic [9], such as a GP-like method [10], responsible for generating new code for the meta-heuristic and running it for some repetitions on one or more benchmark problems. The statistical result of the runs is returned to the hyper-heuristic as the fitness value. In online evolution, on the other hand, the hyper-heuristic and the meta-heuristic work in co-operation, which means that there is a single meta-heuristic run, not a set of repetitions, and the meta-heuristic's code is evolved during the optimization by the hyper-heuristic. Therefore, while in the offline approach the hyper-heuristic tries to find the best static code to solve a single problem or various problems, in the online approach the code is evolved for the particular problem in a particular run. Given that DSO is an online approach, below we consider only similar research from the literature. Unfortunately, we could find only a few work.

Rashid and Baig [11] proposed PSOGP, an extension of [12] that works online, evolving the code during the optimization. Each particle in the population has its force generating function, which is evolved by GP. This approach gave PSO better exploration abilities by slowing convergence and increasing chances of escaping from local optima.

In a similar approach, Si et al. [13] replaced GP by GDE (Grammatical Differential Evolution [14]) and also achieved better performance than the PSO version used in comparison.

One may also consider as online approaches some adaptive methods such as those proposed in [8], [15]–[17], where several methods or operators are coded and available for selection during the optimization process. A mechanism, usually a probability of success in solution improvement, is used to select the most promising codes. However, the options are hard-coded, that is, the code is immutable.

### IV. EXPERIMENTAL RESULTS

This next section presents a comparison of Drone Squadron Optimization with state-of-the-art evolutionary and swarm optimization methods in solving a set of well-known unconstrained/box-constrained continuous global optimization benchmark functions.

#### A. Benchmark functions

DSO was tested on functions  $f1$  to  $f10$  of the benchmark problem set from CEC' 2005 Special Session in Real Parameter Optimization [18]. Those problems present a diverse set of features such as multimodality, ruggedness, ill-conditioning,

interdependency, among others. In our experiments, all functions were solved in 10 dimensions, with a maximum *total* of 100,000 objective function evaluations *shared* by the teams (it is not per team), and all configurations suggested in [18].

#### B. Implementation and configuration of the algorithms

The experiments were executed on an Intel(R) Xeon(R) X5550 @ 2.67 GHz with 8GB RAM, Linux enterprise 3.14.1-gentoo i686. DSO was implemented in MATLAB R2012b. The information and functions available to the Command Center to generate perturbation schemes are shown below.

- **Departure coordinates:** *CurrentBestCoords*, *PermutedCurrentBestCoords*, *PBestCurrentBestCoords*, Multivariate normal sampling (MVNS): new random solutions sampled using the average and covariance matrix of the *p-Best* solutions found, *Opposition(CurrentBestCoords)* as in [19].
- **Other terminals used in Offset:**  $matInterval_D = (\bar{UB}_D - \bar{LB}_D)$ ,  $C1$ ,  $C2$ ,  $C3$ ,  $U(0, 1)$ ,  $U(0.5, 1)$ ,  $G(0, 1)$ ,  $abs(G(0.5, 0.1))$ ,  $abs(G(0, 0.01))$ ,  $Step(CBC) = \sigma * G(0, 1)_{N,D} * matInterval_D * U(0, 0.5)$  as used in [20],  $Shift = TmC - CBC$ , and  $GBC$ .
- **Functions:** *abs*, *neg*, *plus*, *times*, *sub*, *protected division*, *average\_two\_args*.

DSO configuration is shown in Table I and was empirically chosen after a few runs with distinct configurations, but we intend to do a deeper investigation on the influence of the parameters in a future work. *rand/1* is the classical mutation strategy from the Differential Evolution (but not linked to a particular crossover), and MVNS+Step is *inspired* by the CMA-ES technique, but employing only sample generation and step calculation with  $\sigma = 0.04 \times \mu_{eff} \times ||\mu||$ . This formula is from the CMA-ES author's source code and was not tuned to be used in DSO.

DSO will work with 4 teams of 25 members, giving the overall sample size 100. The sample is not necessarily 100 distinct solutions that will be perturbed to give new 100 solutions as performed by other methods because the *Departure* coordinates may be the same for all teams at some iteration.

#### C. Performance comparison

Results are evaluated over 36 runs (instead of 25 as the other methods) because the method was executed on a cluster and all available computing nodes were used. A successful run presents error lower than 1E-09.

The rank analysis is first done by Friedman rank sum test. Then, if the statistical difference among the methods is detected, one employed the Pairwise posthoc Test for Multiple Comparisons of Mean Rank Sums for Unreplicated Blocked Data (Nemenyi-Test) to identify differences between DSO (as the control method) and each method used in the comparison. We assumed a significance level  $\alpha = 0.05$ .

Table I  
RUN AND EVOLUTIONARY PARAMETER VALUES FOR DSO TO SOLVE  
CEC'2005 PROBLEMS.

Parameter	Value
C1, C2, C3	0.5, 0.4, 0.9
Teams	4
Reference perturbations	<i>rand</i> /1 using C1 as F, MVNS+Step every iteration
Firmware update	1
<i>w</i> (# firmware updated)	<i>rand</i> /1 using C1 as F
Fixed perturbation	No recombination, Binomial recombination [21], Exponential recombination [21]
Recombination	50 iterations
Stagnation	1
Elitism when stagnated	10%
$P_{acc}$	5, 20
tree-size ( $s_{min}$ , $s_{max}$ )	25%
$p\text{-Best}$	$U(0.4, 0.9)$
$CR$	

## D. Results and Discussion

In this section, we evaluate DSO on the first ten benchmark functions of the problem set from CEC' 2005 Special Session in Real Parameter Optimization<sup>4</sup>. We focused on the dimension  $D = 10$  as investigated in [22].

Descriptive statistics of DSO runs are shown in Table II. DSO failed for functions f8 and f10 while showed poor performance for functions f3 and f7. The minimum median achieved values suggest that DSO was unable to perform local optimization for these functions as it could not reach the desired precision. Also, mean values are much higher than median values for several functions, meaning outlier failures. This behavior was not expected given the stagnation and escaping procedures employed in the method. Hence, more investigation is necessary to improve this aspect of the method.

Table II  
DESCRIPTIVE STATISTICS OF DSO RUNS OBTAINED IN CEC'2005  
SPECIAL SESSION IN DIMENSION  $D = 10$ .

Fun	Min	Median	Max	Mean	Std.Dev	SR
<b>f1</b>	0.00E+00	0.00E+00	4.08E-19	1.13E-20	6.79E-20	1
<b>f2</b>	7.09E-21	4.06E-16	6.02E-14	4.03E-15	1.19E-14	1
<b>f3</b>	1.90E-07	4.99E-02	2.41E+03	7.70E+01	4.03E+02	0.028
<b>f4</b>	9.39E-17	5.24E-14	1.51E-10	5.72E-12	2.57E-11	1
<b>f5</b>	0.00E+00	0.00E+00	2.15E-01	5.97E-03	3.58E-02	0.944
<b>f6</b>	1.04E-19	4.47E-12	3.99E+00	3.32E-01	1.12E+00	0.917
<b>f7</b>	7.40E-03	5.66E-02	1.57E-01	6.53E-02	3.43E-02	0.028
<b>f8</b>	2.00E+01	2.00E+01	2.05E+01	2.01E+01	1.06E-01	0
<b>f9</b>	0.00E+00	0.00E+00	1.99E+00	4.70E-01	6.93E-01	0.639
<b>f10</b>	2.98E+00	1.34E+01	2.79E+01	1.37E+01	6.16E+00	0

The results shown in Table III are compared with those from Table 13 in [22], which considers only the average error for each problem. The optimization methods in the comparison are: Hybrid Real-Coded Genetic Algorithm with Female and Male Differentiation (BLX-GL50 [23]), Real-Coded Memetic Algorithm (BLX-MA [24]), Cooperative Evolution EA (CoEVO [25]), canonical Differential Evolution

<sup>4</sup>We intend to test all 25 benchmark functions and other problem sizes in a future work.

Table III  
AVERAGE ERROR OBTAINED IN CEC'2005 SPECIAL SESSION IN  
DIMENSION  $D = 10$ . RESULTS *worse* THAN THOSE OF DSO ARE IN ITALIC  
AND BOLD-FACE.

Algorithm	f1	f2	f3	f4	f5
DSO	1.000E-09	1.000E-09	7.697E+01	1.000E-09	5.968E-03
BLX-GL50	1.000E-09	1.000E-09	<b>5.705E+02</b>	1.000E-09	1.000E-09
BLX-MA	1.000E-09	1.000E-09	<b>4.771E+04</b>	<b>1.997E-08</b>	<b>2.124E-02</b>
CoEVO	1.000E-09	1.000E-09	1.000E-09	1.000E-09	<b>2.133E+00</b>
DE	1.000E-09	1.000E-09	1.940E-06	1.000E-09	1.000E-09
DMS-L-PSO	1.000E-09	1.000E-09	1.000E-09	<b>1.885E-03</b>	1.138E-06
EDA	1.000E-09	1.000E-09	2.121E+01	1.000E-09	1.000E-09
G-CMA-ES	1.000E-09	1.000E-09	1.000E-09	1.000E-09	1.000E-09
K-PCX	1.000E-09	1.000E-09	4.150E-01	<b>7.940E-07</b>	<b>4.850E+01</b>
L-CMA-ES	1.000E-09	1.000E-09	1.000E-09	<b>1.760E+06</b>	1.000E-09
L-SaDE	1.000E-09	1.000E-09	1.672E-02	<b>1.418E-05</b>	<b>1.200E-02</b>
SPC-PNX	1.000E-09	1.000E-09	<b>1.081E+05</b>	1.000E-09	1.000E-09
Algorithm	f6	f7	f8	f9	f10
DSO	3.322E-01	6.526E-02	2.005E+01	4.698E-01	1.365E+01
BLX-GL50	1.000E-09	1.172E-02	<b>2.035E+01</b>	<b>1.154E+00</b>	4.975E+00
BLX-MA	<b>1.490E+00</b>	<b>1.971E-01</b>	<b>2.019E+01</b>	4.379E-01	5.643E+00
CoEVO	<b>1.246E+01</b>	3.705E-02	<b>2.027E+01</b>	<b>1.919E+01</b>	<b>2.677E+01</b>
DE	1.590E-01	<b>1.460E-01</b>	<b>2.040E+01</b>	<b>9.550E-01</b>	1.250E+01
DMS-L-PSO	6.892E-08	4.519E-02	2.000E+01	1.000E-09	3.622E+00
EDA	4.182E-02	<b>4.205E-01</b>	<b>2.034E+01</b>	<b>5.418E+00</b>	5.289E+00
G-CMA-ES	1.000E-09	1.000E-09	2.000E+01	2.390E-01	7.960E-02
K-PCX	4.780E-01	<b>2.310E-01</b>	2.000E+01	1.190E-01	2.390E-01
L-CMA-ES	1.000E-09	1.000E-09	2.000E+01	<b>4.490E+01</b>	<b>4.080E+01</b>
L-SaDE	1.199E-08	2.000E-02	2.000E+01	1.000E-09	4.969E+00
SPC-PNX	<b>1.891E+01</b>	<b>8.261E-02</b>	<b>2.099E+01</b>	<b>4.020E+00</b>	7.304E+00

(DE [26]), Dynamic multi-swarm particle swarm optimizer with local search (DMS-L-PSO [27]), Estimation of Distribution Algorithm (EDA [28]), Covariance Matrix Evolution Strategy and Restarting method (G-CMA-ES [29]), Steady-State Evolutionary Algorithm (K-PCX [30]), Covariance Matrix Evolution Strategy Improved with Local Search (L-CMA-ES [31]), Self-adaptive differential evolution algorithm for numerical optimization (L-SaDE [7]), and Steady-State Genetic Algorithm (SPC-PNX [32]).

Friedman rank sum test was performed to compare the 12 techniques' averages. As the Friedman test indicates significant difference ( $\chi^2(11) = 20.6765$ ,  $p = 3.688E-02 < \alpha$ ), it is meaningful to conduct multiple comparisons in order to identify differences between the techniques on the 10 problems. For such purpose, Table IV presents the comparisons regarding *win*, *tie*, and *loss*, as well as Nemenyi's posthoc  $p$ -values.

On the one hand, with respect to the W/T/L values, DSO was better than BLX-MA, CoEVO, EDA, K-PCX, and SPC-PNX. On the other hand, as can be noticed by the  $p$ -values, no significant differences ( $p < \alpha$ ) were detected by the posthoc analysis for  $\alpha = 0.05$ , even though G-CMA-ES and DMS-L-PSO won 7 out of 10 times (lower average error), and DSO won 6 times when compared to BLX-MA. Therefore, one may conclude that DSO is as good as the methods selected for that Special Session, although it is based on an construction mechanism which sometimes generates perturbation schemes that give low quality solutions and waste precious evaluations, while other methods are hand-made specialized developments. Thus, this characteristic is a good opportunity of improving DSO by trying to learn to avoid such poor schemes.

Processing time cannot be compared as not all methods have source code freely available online. Nevertheless, DSO was developed in Matlab and requires parsing and executing the

Table IV  
COMPARISON OF THE AVERAGES: DSO VERSUS OTHER METHODS (WINS/TIES/LOSSES).  $p$ -VALUE WAS CALCULATED BY NEMENYI'S POST-HOC TEST.

	BLX-GL50	BLX-MA	CoEVO	DE	DMS-L-PSO	EDA
W/T/L	3/3/4	6/2/2	5/3/2	3/3/4	1/2/7	3/3/4
$p$ -value	1.00	1.00	1.00	1.00	0.99	1.00
	G-CMA-ES	K-PCX	L-CMA-ES	L-SaDE	SPC-PNX	
W/T/L	0/3/7	4/2/4	3/2/5	2/2/6	5/3/2	
$p$ -value	0.50	1.00	1.00	1.00	1.00	

perturbation source code, which gives a much longer running time.

## V. FURTHER DISCUSSION

In this section, we discuss some aspects of DSO that could raise questions and then clarify them. First of all, DSO employs ideas of some distinct frameworks, thus it does not completely fits to a particular one.

DSO can adapt its parameters to produce better performance during the optimization process, as similar adaptive techniques [7], [17], [33], [34]. However, DSO does more than that.

In DSO, the perturbation operator is also modified, not just the parameters. In this case, DSO can be seen as a generative hyper-heuristic approach [9], where code is evolved instead of parameters. However, in a standard hyper-heuristic application, the generation of codes is the main procedure, while in DSO it is a component of the method. Thus, DSO is a hybrid approach with co-evolution of solutions to the problem being optimized and the procedures to optimize it.

DSO has independent teams operating on current solutions, which is similar to distributed EAs (dEAs) [35]–[37]. However, most dEAs employ the same algorithm on all subpopulations (either with the same or different control parameters). On the other hand, in DSO distinct teams may use the same *Departure* coordinates (same set of solutions - population), but with distinct ways of calculating the *Offset*. At other period of the search, the teams may have the same *Offset* formulation, but distinct *Departures*. Given that the teams may have distinct perturbation and recombination procedures, resulting in distinguishable behaviors, they could be seen as distinct algorithms.

In dEAs, the parallel algorithms have their own populations searching distinct regions of the search-space, and a migration procedure is adopted to try to escape from local optima and to speed-up search. On the other hand, the teams in DSO are not sub-populations; hence no migration is required. Another important argument why migration is not used is because DSO's teams are neither directly co-evolving nor cooperating. In fact, the selection mechanism considers the solutions from all teams; therefore they are primarily competing to each other, even though all teams may improve the same shared information. Nevertheless, a comparison between competition and cooperation is out of the scope of this paper.

By using distinct algorithms at the same time, DSO could be a portfolio [15] of evolving (adapting) evolutionary processes. Most differences for dEAs can be applied to the portfolio

framework, but in portfolios and other similar methods, such as AMALGAM [8] and A-Teams [38], distinct populations use distinct algorithms. However, again, these algorithms are hard-coded and are not improved during the optimization.

Finally, it is important to recall that DSO's teams are semi-autonomous; they must obey the Command Center. The Command Center can upgrade the firmwares, select the solutions found by the teams, change the amount of teams and drones in each team, restart the solutions, perform analyzes of the solutions, start a local-search, among other possibilities.

Regarding computation time, the current DSO implemented in Matlab is certainly slower than all methods used in comparison due to the fact it generates code during the optimization process, and that new code has to be evaluated (parsed and compiled) to give the desired results. Once compiled, the perturbation scheme is as fast as hard-coded functions. However, if the perturbation scheme is often changed, then a slowdown is clearly noticeable.

## VI. CONCLUSIONS AND FUTURE WORK

The Drone Squadron Optimization (DSO) introduced in this paper is a novel self-adaptive method for global numerical optimization. It is a population-based search algorithm, but not nature-inspired. Because it is artifact-inspired, it was developed to be a very flexible technique, allowing the insertion and removal of components without disrupting the original concept. For instance, DSO may or may not apply recombination, and any kind of data recombination is permitted as there is no biology concept involved. DSO is self-adaptive from its first version, because this is the role played by the Command Center.

DSO was evaluated on many well-known continuous benchmark problems and compared to many state-of-the-art methods. After the hypothesis test, no significant difference was found, suggesting that DSO showed competitive performance. However, it is important to remember that DSO is a technique that evolves itself during the optimization, meaning that inefficient perturbation schemes will be generated and that a substantial amount of objective function evaluations will be wasted. This is, for now, a critical weakness of the methodology. As future work we intend to reduce the occurrence of those schemes by adding learning mechanisms to DSO.

The DSO method introduced here is a naïve implementation, using random distributions to generate scalings, to select methods for recombination and bound correction, to select lower-quality solutions when stagnation occurs, to update the



firmware, among other aspects. Questions that arise regarding the configuration and performance of the DSO algorithm require future work, but the promising results indicate that the current version is competitive with state-of-the-art methods.

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